RELATIONS BETWEEN THE SPECTRA AND OTHER CHARACTERISTICS OF THE STARS.

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(*Read April 20, 1912.*)

To the student of the stars, who attempts to arrange our existing knowledge in such a manner that some light may be thrown upon the problems connected with stellar evolution, the spectral classification developed at Harvard is of vital importance.

In such investigations, we must deal, if possible, not with single instances, but with representative averages for groups of stars. But really representative averages are often much harder to obtain than might be supposed. Consider, for example, the actual brightness of the stars. We can find this only when we know the distance of the star—and out of the hundreds of thousands of stars which have been catalogued, we know the distance of barely five hundred. But even if we knew the exact distances of the 6,000 or more stars which are visible to the naked eye, we would not have a fair sample of the general run of stars. To explain how this may happen, let us suppose that there were only two kinds of stars, one equal to the sun in brightness, and the other 100 times as bright as the sun, and that these were distributed uniformly through space, in the proportion of 100 stars of the fainter kind for every one of the brighter. To be visible to the naked eye, a star of the fainter sort must lie within about 55 light-years from the sun; but all the stars of the brighter kind which lay within 550 light-years would be visible. We would therefore be searching for these stars throughout a region of space whose volume was 1,000 times greater than that to which our method of selection limited us in picking out the fainter ones, and our list of naked-eye stars would consequently contain ten stars of the brighter kind to every one of the fainter—though if we could select instead the stars contained in a given region of space, we would find the disparity to be 100 to 1 the other way.

It is therefore a fortunate circumstance that the stars whose distances have been measured have for the most part been chosen, not on account of apparent brightness, but because of relatively rapid proper-motion—which is found by experience to be a fairly good indication of actual nearness to our system. These stars, therefore, represent mainly the sun's nearer neighbors, without such an egregious discrimination in favor of stars of great actual brightness as we have seen must occur if we choose our stars by apparent brightness alone. Some traces of this discrimination will still be unavoidable, for our knowledge of the proper-motions of the fainter stars is still imperfect, and stops short at a little below the ninth magnitude.

In addition to the stars whose parallax has been directly observed, we have data for many more, which belong to clusters whose distances have been found by combining data regarding their proper-motions and radial velocities. In this case too the absence of proper-motion data (which decide whether or not a star really belongs to the cluster) prevents us from obtaining information about stars fainter than a certain limit; but otherwise our knowledge is probably fairly complete.

In the present discussion of the relation between the spectral type and the real brightness of the stars, those directly measured parallaxes have been employed which are confirmed by the work of two or more observers, and also a few results obtained by single observers whose work is known to be of high accuracy, and free from sensible systematic errors. To these have been added the members of the Hyades, the Ursa Major group, the "61 Cygni group" and the moving cluster in Scorpius discovered independently by Kapteyn, Eddington, and Benjamin Boss. The spectra of a very large number of these stars have been determined at Harvard especially for this investigation, and the writer takes pleasure in expressing his most hearty thanks to Professor Pickering and Miss Cannon for this generous and invaluable aid.

The actual brightness of the stars may best be expressed by means of their "absolute magnitudes"—*i. e.*, the stellar magnitudes which they would appear to have if each star was brought to the standard distance of 32 light-years (corresponding to a parallax of 0″.10). The absolute magnitude of the sun on this scale is about 4.7.

On plotting these absolute magnitudes against the spectral types it becomes immediately evident that most of the stars belong to a series in which the fainter members are redder than the brighter, while a few outstanding stars of each spectral class greatly exceed in brightness those belonging to this series (except for class B, all of whose stars are very bright). The existence of these two series was first pointed out by Hertzsprung,[1] who has called them by the very convenient names of "giant" and "dwarf" stars—the former being of course the brighter.

With the large amount of material now available, especially for the dwarf stars, the results derived from the stars with directly measured parallaxes and from those in the clusters are in striking agreement, as is shown in Table I.

TABLE I.

MEAN ABSOLUTE MAGNITUDES OF DWARF STARS.

Spectrum.	Stars with Measured Parallaxes.			Stars in Clusters.			Formula.
	Number.	Abs. Mag.	Light.	Number.	Abs. Mag.	Light.	Abs. Mag.
B–B3	14	−1.7	380.	21	−1.2	240.	−1.0
B5–B9				8	0.3	60.	0.1
A	6	0.7	42.	13	0.5	50.	0.5
A2–A5	6	1.9	14.	26	1.7	17.	1.6
F	6	4.0	2.0	18	3.0	5.0	2.7
F5	7	3.6	2.9	7	3.3	3.7	3.8
F8	7	4.5	1.3	7	4.5	1.3	4.5
G	21	5.4	0.55	19	4.9	0.87	4.9
G5	11	5.6	0.46	9	5.5	0.50	6.0
K	16	6.6	0.18	10	6.3	0.24	7.1
K5	16	8.3	0.04	7	(6.6)	(0.18)	8.2
Ma	10	9.7	0.01				9.3

In the above table, the quantity given under the heading "Absolute Magnitude" is the mean of the individual values derived from the observed magnitude and parallax of each star in the corre-

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[1] *Zeitschrift für wissenschaftliche Photographie,* Bd. V., p. 86, 1907.

sponding group (giving half weight to a few stars of relatively uncertain parallax or spectrum)—except for the stars of spectrum B with directly measured parallaxes. In this case the parallaxes are so small that a reliable value could be obtained only by taking the mean of the observed magnitudes and parallaxes for the whole group. These stars are of much greater apparent brightness than most of those of class B, and their actual brightness may be greater than the average for the class. No similar error of sampling need be suspected in other cases, except for the faintest stars in the clusters, where it is obvious in going over the lists that only a few of the brightest stars of class K5 are above the limit of magnitude at which our catalogues of stars belonging to the clusters stop, and probable that some of the fainter stars of class K are also excluded.

With the exceptions just explained, the results of the two independent determinations from the measured parallaxes and the clusters are in remarkably good arrangement, considering the small numbers of stars in many of the groups. The absolute magnitudes of stars of the same spectral class in different clusters are in equally good agreement. The relation between absolute magnitude and spectral type appears therefore to be independent of the origin of the particular star or group of stars under consideration.

This relation seems to be very nearly linear, as is shown by the last column of Table I., which gives for each spectral type an absolute magnitude computed by the formula

$$\text{Abs. Mag.} = 0.5 + 2.2\ (\text{Sp.} - \text{A}),$$

in which spectrum B is to be counted as 0, A as 1, F as 2, etc. It is of interest in this connection to remember that the difference of the visual and photographic magnitudes of the stars is also nearly a linear function of the spectral type.

The individual stars of each spectral class are remarkably similar in real brightness. Excluding those for which the parallax or spectrum is considerably uncertain, there remain in all 218 stars. Of these only 11, or 5 per cent. of the whole, differ more than two magnitudes in absolute brightness from the value given by the formula for the corresponding spectral class, while 150, or 69 per cent., have absolute magnitudes within one magnitude of the computed value.

The series of stars so far discussed does not however comprise all those in the heavens. Most of the stars of the first magnitude have small parallaxes, and are of great absolute brightness; and a study of proper-motions shows the same to be true of the naked-eye stars in general. It follows that there exists another series of stars, of great brightness, differing relatively little from one spectral class to another. These "giant" stars can be seen at enormous distances, and consequently form a wholly disproportionate part of the stars visible to the naked eye, as has been explained above. The illustration there given greatly understates the actual situation for the redder stars. The dwarf stars of class M, for example, are so faint that not one of them is visible to the naked eye (though one of them is the second nearest star in the heavens), and so the naked-eye stars of this class are all "giants."

Relatively few of these giant stars are near enough for reliable measures of parallax, and even for these it is safer to take the mean observed parallaxes and magnitudes of groups of stars, to diminish the effect of errors of observation. Confining ourselves as before to parallaxes determined by two or more observers, or by observers of high accuracy, the existing data may be summarized as follows.

TABLE II.

Mean Absolute Magnitudes of Giant Stars.

Spectrum.	Stars with Measured Parallaxes.					Stars in Clusters.		
	Number.	Mean Obs'd. Mag.	Mean Parallax.	Corresponding.		Number.	Mean Abs. Mag.	Light.
				Abs. Mag.	Light.			
B	14	2.2	0″.017	−1.7	380	21	−1.2	240
A to G	5	0.8	0 .033	−1.6	350			
K	7	1.9	0 .032	−0.5	130	11	0.3	60
K5 and M	4	1.4	0 .047	−0.2	100	1	−3.8	2,600 (Antares)

The stars of class B are repeated here, since they may be regarded as belonging to either series.

Here again the stars whose parallaxes have been directly measured have been selected on account of their apparent brightness, and are probably brighter than the average of all the giant stars. Individual stars are in some cases still brighter; for example, Antares, which is clearly shown by its proper-motion and radial velocity to belong to the moving cluster in Scorpius, with a parallax of about 0″.010, and hence must be fully 2,500 times as bright as the sun. Canopus and Rigel, whose parallaxes are too small to measure, are probably equally bright or brighter. Whether there are many more stars of such enormous luminosity, and, in general, whether the giant stars of a given spectral class resemble one another in brightness as closely as the dwarf stars do, cannot be determined from existing data, at least of the kind considered here.

The giant and dwarf stars are fully separated only among the spectral classes which follow the solar type in the Harvard classification. For class A the two series are intermingled, and even for class F, where the average brightness of the two differs by four magnitudes, it would be difficult to say whether a star of absolute magnitude near 1.0 should be regarded as an unusually faint giant star or an unusually bright dwarf. From class G onward, the reality of the separation into two groups is unequivocally indicated by the observational data.

As a practical application of the principles just developed, we may consider the question of the distance of the Pleiades, a problem so far practically unsolved.

The spectra of the fainter stars which are known to belong to the cluster have been determined at Harvard, through the kindness of Professor Pickering and Miss Cannon. They exhibit a very conspicuous relation between apparent magnitude and spectral type, as is shown in the first four columns of Table III.

These stars evidently belong to the series of dwarf stars. The relative brightness of the different spectral classes is in good agreement with that previously found, except that the stars of class B5 in the Pleiades appear to exceed those of class A in brightness as much as those of class B0 to B3 do among the stars previously studied.

TABLE III.

STARS BELONGING TO THE PLEIADES.

Spectrum.	No.	Mean Obs'd. Magnitude.	Group Means.	Abs. Mag. of Stars in Other Clusters.	Difference.	Hypothetical Parallax.
B5	6	4.14	4.7	−0.5	5.2	0″.0091
B8	6	5.35				
B9	7	6.85	7.0	0.5	6.5	0.0050
A0	8	7.08				
A2	5	7.97	8.0	1.7	6.3	0.0055
A5	5	8.09				
F and F5	5	8.88	8.9	2.5	6.4	0.0052
G	3	9.43	9.4	(3.6)	(5.8)	(0.0066)

The fifth column in the table gives the mean absolute magnitudes previously found for stars of similar spectral type in other clusters (choosing the brighter half of those of class F, and a few of the brightest stars of class G, since it is evident that the limitation to stars above a given magnitude compels a similar choice in the Pleiades). From the differences between the observed and absolute magnitudes, we may compute the distances to which a group of stars similar to those already studied must be removed in order to appear equal in average brightness to the stars of the same spectral class in the Pleiades. The hypothetical parallaxes so obtained are given in the last column of the table. With the exception of that derived from class B, they are in extraordinary agreement. If they are treated as independent determinations of the parallax, of equal weight, the resulting mean is 0.″0063 ± 0″.0006, corresponding to a distance of 500 light-years.

This estimate of the distance of the Pleiades depends upon the assumption that, when we find in this cluster the same relation between the relative brightness of the stars of different spectral classes that exists elsewhere, wherever the real brightness of the stars can be investigated, the absolute brightness for each spectral class is also approximately the same as elsewhere. This assumption is made decidedly probable by the fact that it undoubtedly holds true for the stars of the four clusters whose distances are known, and for more than 100 other stars not belonging to clusters, with no serious exceptions. It should however be remembered that no account has been taken of possible absorption of light in space, and that there are unusually few very faint stars in the region of the Pleiades, which has been explained as the result of partial opacity of the nebulosity surrounding the cluster. Some of this nebulosity presumably lies between us and the stars of the cluster, and cuts off a part of their light, which would make the distance computed on the assumption that there was no absorption come out too great. If such absorption exists, it should be possible to determine its amount, and allow for it.

It is of obvious interest to inquire in what other respects besides brightness the giant and dwarf stars of the same spectral class differ from one another. One line of approach is furnished by the visual binary stars. It is well known that, when the orbital elements and apparent brightness of a binary pair are given, we can find what Professor Young calls the "candle-power per ton"—more exactly, the ratio L^3/M^2 where L is the combined light of the pair, and M the combined mass—without knowing the parallax. The writer has recently shown[2] that this principle can be extended by simple statistical methods to the stars known to be physically connected whose orbits cannot yet be computed. In this way about 350 stars have been investigated, and it is found that they fall into two series, similar in all respects to the giant and dwarf stars,—one marked by high luminosity per unit of mass, nearly the same for all spectral classes, and the other by small luminosity per unit of mass, diminishing very rapidly for the redder stars. By means of the parallactic motions of these groups of stars, an approximate estimate can be made of their distances, absolute magnitudes and masses, with results which may be summarized as follows.

TABLE IV.

MEAN ABSOLUTE MAGNITUDES AND MASSES OF BINARY STARS.

Spectrum.	Giant Stars.			Spectrum.	Dwarf Stars.		
	Number.	Abs. Mag.	Mass.		Number.	Abs. Mag.	Mass.
B	44	−1.0	7	A–A5	57	1.2	5.
A–A5	52	−1.0	13	F–F5	61	3.0	3.5
F–G	26	−0.6	8	F8–G	32	4.7	1.4
K–M	38	−0.5	8	G5–K	26	6.3	0.5
				K5–M	8[3]	9.0	0.8

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[2] *Science,* N. S., vol. 34, p. 524, 1910.

[3] From directly measured parallaxes.

The mean absolute magnitudes agree almost perfectly with those already derived for other groups of stars, showing that we have come again upon just the same giant and dwarf stars in still a different way. The computed masses, although subject to errors which may in some cases be as great as 50 per cent., show that the brighter stars are more massive than the fainter, but that the differences in mass are small compared with those in luminosity.

We may go farther with the aid of the information regarding stellar densities which can be obtained from the eclipsing variables, which are mostly of classes B and A. The average density of the eclipsing variables of class B is about one seventh of the Sun's density. We may therefore estimate that a typical star of the class, with seven times the sun's mass, is between three and four times the sun's diameter, and has about 15 times his superficial area. But we have already found that such a star, on the average, gives out more than 200 times as much light as the sun. Hence its surface brightness must be about 15 times as great as that of the sun. In the same way it is found that stars of class A must exceed the sun five-fold in surface intensity. On the other hand, the faint stars of classes K5 and M give off on the average about 1/100 of the sun's light, with masses exceeding half the sun's. Even if they were as dense as platinum, their surface brightness could not exceed 1/15 that of the sun.

This diminution of surface brightness with increasing redness, which has been proved to exist among the dwarf stars, is in obvious agreement with the hypothesis (now well established on spectroscopic grounds) that the principal cause of the differences between the spectral classes is to be found in differences in the effective surface temperatures of the stars; and the numerical results here obtained are in good agreement with those computed by Planck's formula from the effective temperatures derived by Wilsing and Scheiner from their study of the distribution of energy in the visible spectrum.

That the same law of diminution of the surface brightness with increasing redness holds true among the giant stars is highly probable, for giant and dwarf stars of the same spectral class are almost exactly alike in color and spectrum. If this is true, the giant stars, which are nearly equal in mass and brightness for all spectral types, must decrease very rapidly in density with increasing redness. If the relative surface brightness of classes B, G, and M is as given above, it is easy to show that the average density of the giant stars of class G must be about 1/40 of those of class B, or about 1/250 of the sun's density, and that the density of the giant stars of class M must average only about 1/15,000 of that of the sun. There is no escape from this conclusion unless we assume that the relation between spectral type and surface brightness is radically different for the giant and dwarf stars, in spite of the practical identity of the lines in their spectra and the distribution of energy in the continuous background.

The nature of the connection which class B forms between the two series is now evident. If all the stars are arranged in order of increasing density, the series begins with the giant stars of class M, runs through the giant stars to class B, and then, with still increasing density, through the dwarf stars, past those which so closely resemble the sun, to the faint red stars.

This arrangement is in striking accordance with the theoretical behavior which a mass of gas, of stellar order of magnitude, might be expected to exhibit if left to its own gravitation and radiation, at a very low initial density. While the density remains low, the ordinary "gas laws" will be very approximately obeyed, and, in accordance with Lane's law, the temperature must rise in order that the body may remain in equilibrium as its radius diminishes. At first the central temperature increases in inverse ratio to the radius, and that of the radiating layers near the surface also rises, though more slowly (because we see less deeply into the star as it becomes denser). As the density of the gas increases further, it must become more difficultly compressible than the simple gas laws indicate; and internal equilibrium can be maintained with a smaller rise of temperature after contraction. The temperature will finally reach a maximum, and the star, now very dense, will cool at last almost like a solid body, but more slowly, for contraction will still take place to some extent, and supply heat to replace much of that lost by radiation.

The highest temperature will be attained at a density for which the departures from the gas laws are already considerable, but probably long before the density becomes as great as that of water.

The density of the stars of classes B and A (which all lines of evidence show to be the hottest) is actually found to average about one fifth that of water, that is, of just the order of magnitude predicted by this theory. It appears therefore to be a good working hypothesis that the giant and dwarf stars represent different stages in stellar evolution, the former, of great brightness and low density, being stars effectively young, growing hotter and whiter; while the latter, of small brightness and high density, are relatively old stars, past their prime, and growing colder and redder. The stars of class B, and probably many of those of class A as well, are in the prime of life, and form the connecting link between the two kinds of red stars.

Princeton University Observatory,
May 25, 1912.